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Theory of Time Dependent Mechanical Breakdown of Oriented Media*

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A phenomenological theory of the time dependent nature of mechanical breakdown phenomena is presented for homogeneous oriented and stressed solids. On the basis of considering the kinetic breaking and forming processes in the microscopic molecular structure, a general theory is formulated and analyzed. The solution to this formulation gives not only reasonable results comparable to available experimental findings but also yields new information which may or may not be able to be verified experimentally. In general it is found that for a wide range of large applied tensile stresses, the fracture strength is almost linearly related with logarithm of time. For small stresses, however, the time-to-break required approaches to infinity.

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The macroscopic mechanical strength of a solid is intimately affected by microscopic structural variations. This is particularly true for solids having pronounced time-dependent properties. Ultimate strength is one such property which can be considered on the basis of a kinetic concept in the fracturing processes. The variation of molecular forces must be considered if a realistic physical behavior is to be obtained. With this in mind, the mathematical model which can be used for analysis, is a matrix of oriented elements representing molecular forces, embedded in an arbitrary domain. For simplicity, consider that the system is composed of a large number of identical elements. For such a system, the state of stress in the vicinity of a point in the solid may be represented by determining the stress tensor $\sigma_{ij} (i, j = 1,2,3)$ at that point[1]

$$\sigma_{ij} = \int \rho f \psi s_i s_j d\omega \tag{1}$$

where ρ is the density of probability distribution function of orientation, ψ is the axial stress acting on each element, and is a function of orientation and time. f is the fraction of unbroken elements and is also a function of orientation and time. s_i and s_j are unit vectors, and $d\omega$ is the infinitesimal solid angle within which the elements are considered to be parallel. In general the time dependent nature can be formulated utilizing the statistical theory of the absolute reaction rate for a given orientation of the elements with respect to the direction of applied stress[1,2]

$$\frac{df}{dt} = K_r (1 - f) - K_b f \tag{2}$$

where

$$K_r = \omega_r e^{-U/RT - \gamma\psi(t)} \tag{3}$$

is the rate of reformation of broken elements and

$$K_b = \omega_b e^{-U/RT + \beta\psi(t)} \tag{4}$$

the rate of rupturing of unbroken elements ω_r and ω_b are respectively the frequencies of motion of the elements with respect to forming and breaking processes. U is activation energy, R is a universal constant and T is absolute temperature. β and γ are positive quantities which modify the energy barrier for the applied stress in the direction of each element.

For a completely oriented system, if the strength of a solid under a simple constant tension, say in 33-direction, is considered then from (1):

$$\psi(t)f(t) = \sigma_{33} = \sigma \tag{5}$$

where σ = constant is the applied simple tension and f is independent of molecular orientation. In this case differentiating (5) and combining with (2), one can obtain the functional relationship between ψ and t.

$$\frac{d\psi}{dt} = -\frac{\psi^2}{\sigma}\left[K_r\left(1 - \frac{\sigma}{\psi}\right) - K_b\frac{\sigma}{\psi}\right] \tag{6}$$

Integration results in the time-to-break

$$t_b = \int\left\{K_b\psi\left[1 + \frac{K_r}{K_b}(1 - \psi/\sigma)\right]\right\}^{-1}d\psi + t_\circ \tag{7}$$

where $t_\circ$ is a constant of integration.

As shown in (2), (3) and (4), K_b and K_r are both functions of $\psi(t)$. The basic concept is describable according to the absolute reaction rate theory. In general the effect of stress on elastic deformation and viscous flow is capable of being related to a comprehensible activation process as a consequence of the movement of the whole or a segment of a molecule, from one equilibrium state to the next. If U is the original potential energy barrier to be crossed between two equilibrium states, the frequency with which such steps occur under the influence of the thermal agitation will be proportional to $e^{-U/RT}$: After a stress $\sigma(t)$ is applied to the oriented system, individual elements are subjected to a stress $\psi(t)$. The energy barrier associated with the breaking process becomes modified to $U/RT - \beta\psi(t)$ in the direction of the applied stress. Whereas in the opposite sense, the energy barrier is modified to $U/RT + \gamma\psi(t)$. As a result, both the rates of modified kinetic processes become different exponential functions of stress. With this information, (7) can be modified to give the following integral for the time-to-break with $t_o = 0$:

$$t_b = e^{U/RT}\int \frac{d\psi}{\psi[\omega_b e^{\beta\psi} + \omega_r e^{-\gamma\psi}(1-\psi/\sigma)]} \tag{8}$$

where the integral is to be evaluated from $\psi = \psi_o$ an initial value of ψ to $\psi = \psi_b$ for which the microscopic element ceases to carry any load. The initial value ψ_o can be expressed through the use of (5) in terms of the applied simple tension σ when the initial value f_o of f is known. Evaluation of the integral (8) will give the required solution. It can be seen that the integration of (8) explicitly in terms of the known functions may not be possible and a numerical evaluation may become a necessity. However (8) can be evaluated explicitly if we restrict to stress ranges within which σ is relatively large. Since f has a bounded variation between 0 and 1, $\psi(t)$ will also be relatively large. Then (8) may be approximated to give

$$t_b = e^{U/RT}\int_{\psi_o}^{\psi_b} \frac{d\psi}{\omega_b \psi e^{\beta\psi}} \tag{9}$$

Integrating we get

$$t_b = \omega_b^{-1} e^{U/RT}\left[\mathrm{Ei}(-\beta\psi_b) - \mathrm{Ei}(-\beta\psi_o)\right] \tag{10}$$

where $\mathrm{Ei}(-x)$ is exponential integral defined by

$$-\mathrm{Ei}(-x) = \int_x^{\infty} y^{-1} e^{-y} dy \tag{11}$$

Substituting $\psi_o = \sigma/f_o$ we obtain

$$t_b = \omega_b^{-1} e^{U/RT}\left[\mathrm{Ei}(-\beta\psi_b) - \mathrm{Ei}(-\beta\sigma/f_o)\right] \tag{12}$$

A variation of t_b v.s. σ is shown on logarithmic scale in Fig. 1 by the solid curve. The maximum limit of the applied stress is indicated by σ_b. If the initial number of unbroken elements is f_o then $\sigma_b = \psi_b/f_o$. There is a considerable region where the curve is almost linear. The lower portion of the curve shown as dotted curve does not characterize the true behavior, however, because the contributions resulted in from the reformation processes are not negligible. In this region, σ is relatively small. There exists a minimum value σ_m for which t_b goes to infinity asymptotically. This value is given by the condition that

$$\infty = \int_{\psi_m}^{\psi_b} \left\{\left[\omega_b e^{\beta\psi} + \omega_r e^{-\gamma\psi}(1-\psi/\sigma_m)\right]\psi\right\}^{-1} d\psi \tag{13}$$

where $\psi_m f_o = \sigma_m$.

If we let σ_r represent approximately the stress at which reformation processes become dominent, then the true curve can be expressed by two integrals:

$$t_b = e^{U/RT}\left[\int_{\sigma_r}^{\psi_b} \frac{d\psi}{\omega_b \psi e^{\beta\psi}} + \int_{\sigma}^{\sigma_r} \frac{d\psi}{\psi[\omega_b e^{\beta\psi} + \omega_r e^{-\gamma\psi}(1-\psi/\sigma)]}\right] \tag{14}$$

If $\sigma_r - \sigma$ is small then the integrand in (14) can be approximated as

$$t_b = \frac{e^{U/RT}}{\omega_b}\left[\mathrm{Ei}(-\beta\psi_b) - \mathrm{Ei}(-\beta\sigma_r) + \int_o^{\sigma_r-\sigma} \frac{d\phi}{(\sigma+\phi)P_n(\phi)}\right] \tag{15}$$

$$(o \leq \psi - \sigma = \phi \leq \sigma_r - \sigma)$$

where $P_n(\phi)$ is a polynomial of n^{th} degree in ϕ. The evaluation of (15) is simple and accuracy can be improved by increasing the number n. Comparison of this analytical result with experimental data has been made. Good agreements have been obtained. Detailed information will be reported elsewhere.

It might be of interest to mention that the initial horizontal portion of the curve in Fig. 1 represents the strength behavior of a medium for an extremely short duration of testing ($<$ one microsecond). Under present available testing capabilities, the experimental strength data lie mostly in the straight line region except for relatively small applied simple stresses.

In addition, on the basis of this model, it is found qualitatively from (10) for a moderate constant applied tension, the logarithm of time-to-break is, for certain region, almost inversely proportionally to the absolute temperature at which the medium is fractured. This is clearly shown in Fig. 2. In the determination of this theoretical curve, β has been considered as directly proportional to the reciprocal of the absolute temperature. This is justified as shown in (3) and (4) both β and γ are likely to be functions of $1/T$ since $\beta\psi(t)$ or $\gamma\psi(t)$ modify the energy barrior in the molecular processes. With this information and some assumed values for constants involved in (10) a qualitative representation of the temperature and time-dependent behavior for fracturing a medium is obtained.

References

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2. C. C. Hsiao, Physics Today 19 No. 3, 49 (1966)

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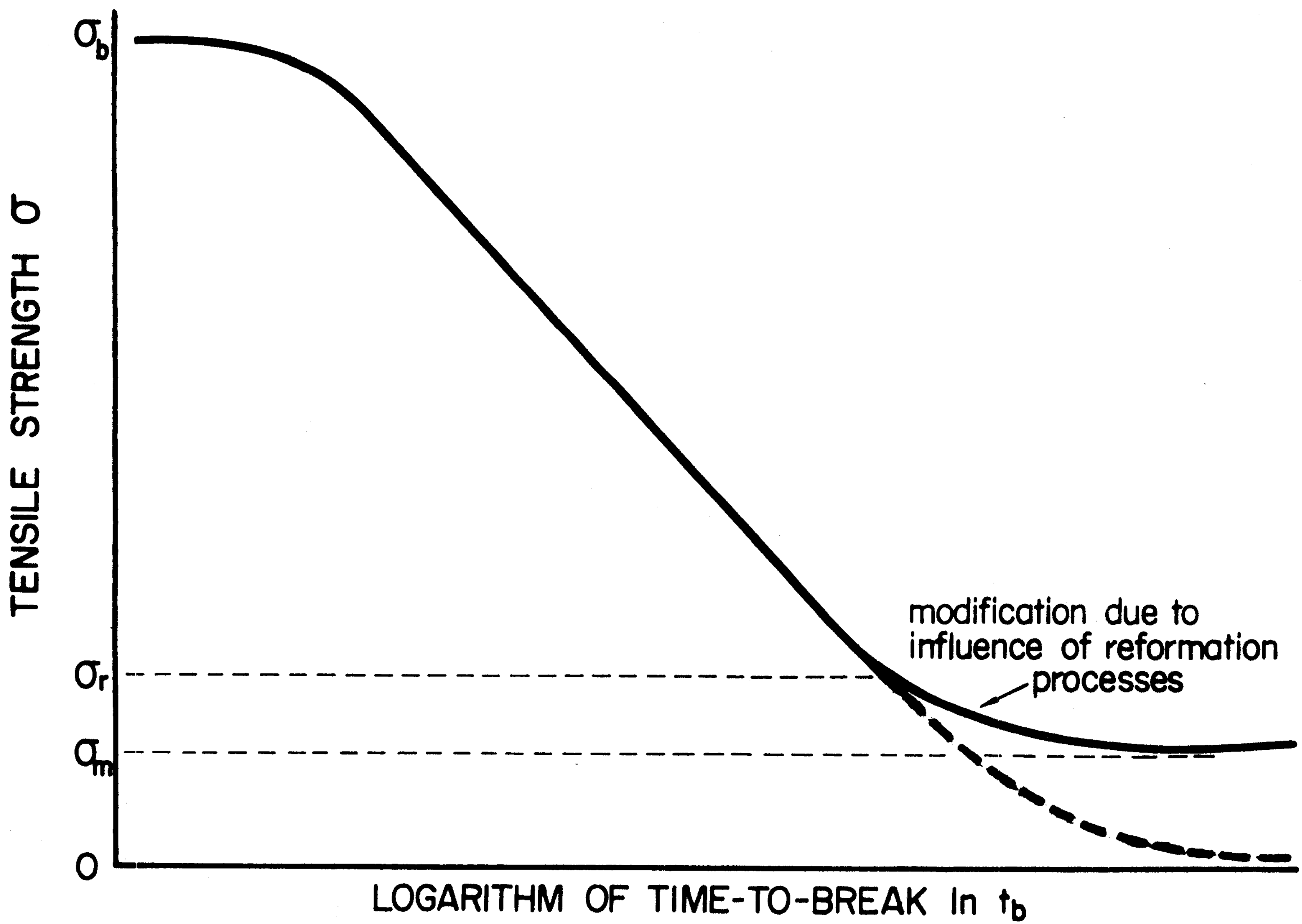

Fig. 1 Time-Dependent Mechanical Breakdown of Oriented Media

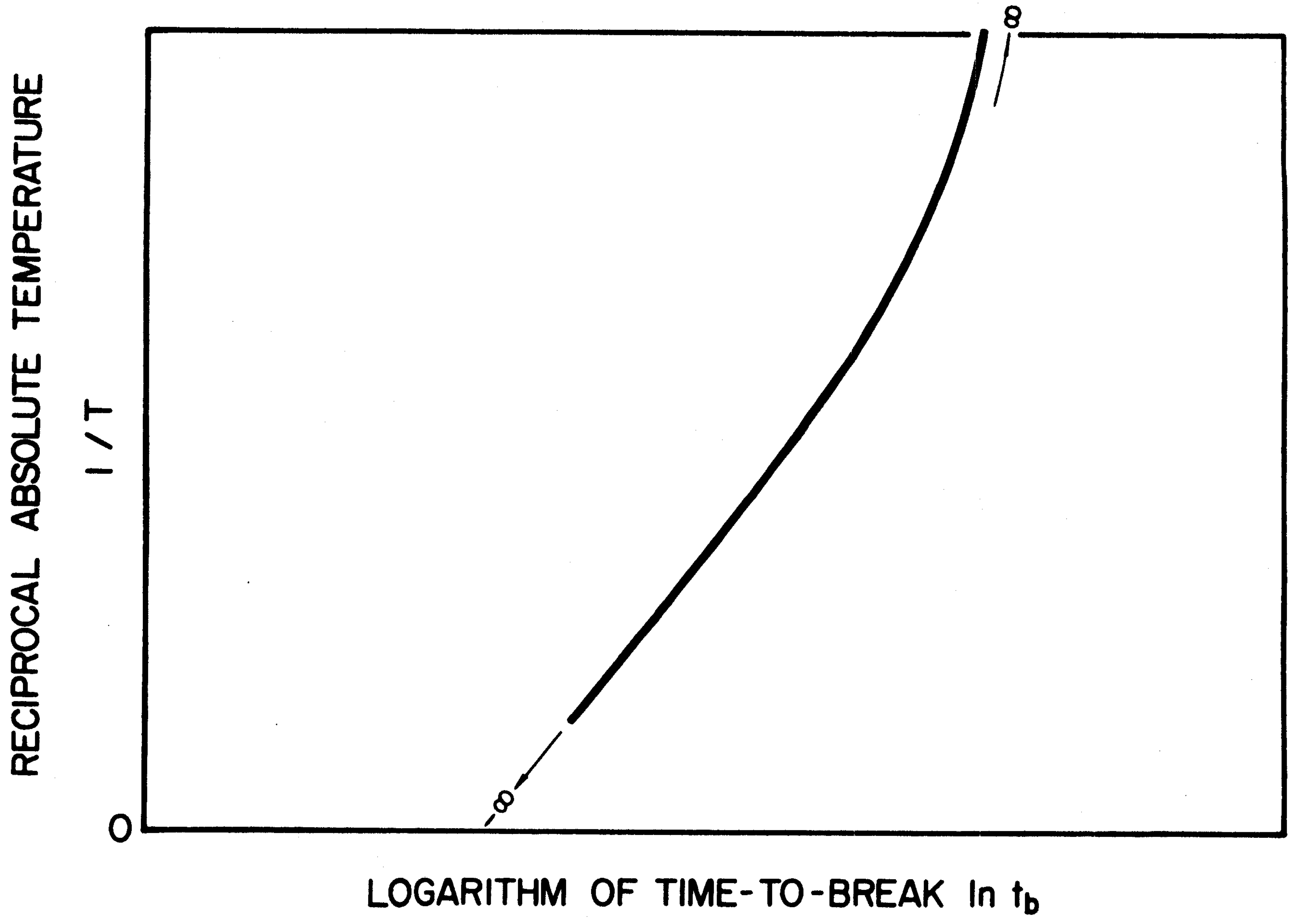

Fig. 2 Temperature and Time Dependent Mechanical Breakdown of Oriented Solids